# Assignment 3

January 26, 2018

1. Let $X_1, ..., X_n \overset{iid}{\sim} U(\theta, \theta+1)$. Show that the minimal sufficient statistic $(X_{(1)}, X_{(n)})$ observed in the class is not complete.

2. Let $X_1, ..., X_n \overset{iid}{\sim} N(\theta, a\theta^2)$ where $a$ is a known constant and $\theta > 0$. Show that the statistic $(\bar{X}, S^2)$ is a sufficient statistic for $\theta$, but the family of distributions is not complete.

3. Let $X$ takes values $0, 1, 2$ with probabilities $p, 3p, 4p$. Determine if the family of distributions of $X$ is complete.

4. Let $X_1, ..., X_n$ be a random sample from the pdf $f_\mu(x) = e^{-(x-\mu)}$, $-\infty < \mu < x < \infty$. Show that $X_{(1)}$ and $S^2$ are independent.

5. Let $X_1, ...X_n \overset{iid}{\sim} Ber(p)$, and define the function $h(p) = P_p(\sum_{i=1}^n X_i > X_{n+1})$, the probability that the sum of first $n$ observations exceeds $(n+1)$ th observation.

   (a) Show that

   $$T(X_1, ..., X_{n+1}) = \begin{cases} 1 \text{ if } \sum_{i=1}^n X_i > X_{n+1} \\ 0 \text{ o.w.} \end{cases}$$

   is an unbiased estimator of $h(p)$.

   (b) Find the best unbiased estimator for $h(p)$.

6. $X_1, ..., X_n \overset{iid}{\sim} Gamma(\alpha, \beta)$, with $\alpha$ known. Find the best unbiased estimator of $1/\beta$.

7. Suppose that $X_1, ..., X_n \overset{iid}{\sim} Ber(p)$.

   (a) Show that variance of $\bar{X}$ attains Cramer-Rao lower bound.

   (b) Find the best unbiased estimator of $p^8$ when $\sum_{i=1}^n X_i > 8$.

8. $X_1, ..., X_n$ be a random sample from a population with p.d.f. $f_\theta(x) = \frac{1}{2\theta}$ for $-\theta < x < \theta$, $\theta > 0$. Find the best unbiased estimator of $\theta$.